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Hilbert Transform and its Applications: A survey

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Abstract—The Hilbert Transform (HT) has globally played an important part in theory of signal processing operation in both continues and digital systems theory because of its relevance to such problem as envelope detection, as well as its use in relating the real and imaginary component, and the magnitude and phase component of spectra.A recent development, proposes an approach to the solution for the nonlinear and non-stationary class of spectrum analysis problems, renowned as Hilbert-Huang Transform (HHT). By using first the Empirical Mode Decomposition (EMD), followed by the HT of the empirical decomposition data.In this survey paper, HT was introduced and its usefulness in the field of signal processing was explained, Then the use of the HHT in signal processing domain was discussed, also an outline to the recent development methods called EMD was introduced, Finally, the application of the HHT was introduced.

Index Terms—Hilbert Transform, Hilbert-Huang Transform, Empirical mode Decomposition.

1 Introduction

Hilbert Transform (HT) have played an important role in signal domain and it also has very large applications in different fields. In HT, if the input signal was choosing to be a sine function then the output will give cosine function, because HT provides a ±90° phase shift to the input signal. So It's like a special phase adjusted filters.

In other transform like Fourier or Wavelet one can change the time domain signal to the frequency domain signals, but in HT domain the operation remains the same in its time or frequency domain [3].

Hilbert–Huang Transform (HHT), it's completely adaptive and analysis data method, had been intend at first by Huang [1] in 1998, it has assured to be sturdy for signal processing and analysis in time series, because of its tremendous popularity. So, it has a wide variety of applications in many fields, compared with others techniques such as wavelet and Fourier analysis. The good feature of HHT is that, it is a completely data-driven, adaptive method, chiefly for processing and analyzing non-stationary and nonlinear signals [1].

The core elements of HHT are Empirical Mode Decomposition (EMD) and HT. The fundamental part of the HHT is the EMD method. The target of EMD is to decompose a signal into a fixed number of Intrinsic Mode Functions (IMFs) with multi-scale modes from smooth to thick and a residue. In [5], By using the EMD method, any sophisticated data set can be decomposed into a fixed and small number of components. These components format almost orthogonal to the original signal.

In order to get more meaningful local information, one must do HT on each IMFs after EMD is done. Because it's ultimately calculate the frequency-energy allocation of a signal from each IMF. Without HT, one will be powerless to get phase information, local frequency and energy [4].

In this survey paper we first explain the HT, especially with signal processing domain and then show its important properties and application, we also describe the HHT in details, and how we can use the EMD with HT to get better result.

2 Hilbert Transform

In the field of signal theory HT is one of the most important operators, it is a linear operator. Given some function u(t), its Hilbert transform, denoted by H(u(t)), is calculated through the integral, as shown in equation (1).

$$H\big(u(t)\big) = \lim_{\epsilon\to 0}\frac{1}{\pi}\int_{|s-t|>\epsilon}\frac{u(s)}{t-s}\,ds \qquad (1)$$

The HT is named by David Hilbert, its first use dates back to 1905. And it was proved by Marcel Riesz in 1928, mainly, the importance of the transform came from its property to extend real functions into analytic functions. This property increases a number of applications, especially in signal theory [6].

One can find c(t) function (companion function), for r(t) function (real function), by using HT. so that: "m(t) = r(t)+ i c(t)" can be from the real line to complex plane (upper half), by doing analytic extend.

HT can be calculate in a slights steps in the area of signal processing: at first, calculate the fast Fourier transform of the particular signal r(t). Second, deny the negative frequencies. At last, find the inverse fast Fourier transform, and the outcome will be a complex-valued signal form a HT pair, consist of real and the imaginary parts.

|m(t)| can be regarded as a slow-varying envelope of r(t). When r(t) is narrow-banded, thus, one can use HT as a way to perform a narrow-band signal in idiom of frequency and

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amplitude modulation.
With HT the domains don't alteration. A time domain function or a frequency domain function remains in the same domain [7].
One of good feature of HT that is a kind of filter, in which the amplitude of the spectral components is remain unaltered, but their phase is altered by 90 degrees, positively or negatively. So, the energy is still unchanged because a phase shift does not change the energy of the signal only amplitude changes can do that [8].

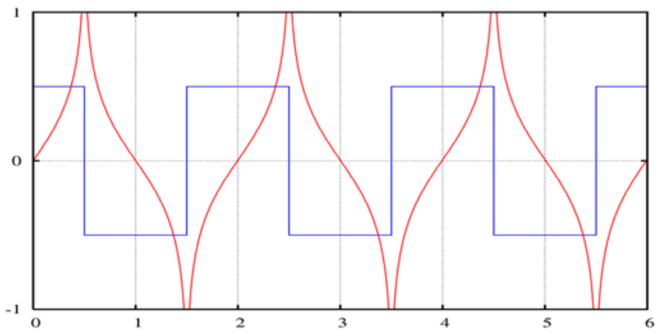

Fig. 1. The Hilbert Transform of a square wave.

2.1 Hilbert Transform Properties

HT has different properties, there are [8]:

1. The HT of a real function is linear.
2. The HT of a HT is the negative of the original function.
3. The HT of the derivative of a function is equivalent to the derivative of the HT of a function.
4. Each function and its HT value are orthogonal.
5. The energy in a real function and its HT are equal.

Example 1. Let y(t) = H(x(t)), y1(t) = H(x1(t)), y2(t) = H(x2(t)) and let a, a1, a2 be some arbitrary constants.Then the HT satisfies the following basic properties [6]:

1. Linearity: H(a1x1(t) + a2x2(t)) = a1H(x1(t)) + a2H(x2(t)).
2. Time shift: H (x (t - a)) = y (t - a).
3. Scaling: H (x (at)) = y (at); a > 0.
4. Time reversal: H (x (-at)) = -y(-at); a > 0.
5. Derivative: H (x' (t)) = y' (t).

3 Hilbert-Huang Transform

From natural phenomena, life science and economic systems, the time series data are mostly nonlinear (is a system in which the output is not directly proportional to the input) and non-stationary (is a system in which it's statistics change over time).

The traditional analysis method like Wavelet and Fourier transform take advantage of the linear and stationary system. The new research tries to use adaptive function bases, [1] produces IMF as adaptive function in the form ofHilbert spectrum expansion. So the HHT is the HT applied to the IMF components [9].

3.1 Intrinsic mode function (IMF)

Huang [1] analyze the requirement of expressive frequency on Hilbert transformation and produced the adaptive function as IMF, which pleased the two condition:

- In the entire data set, the number of zero crossing have to similar or different maximum by one.
- At any point, the average value of envelope defines by local maxima and the envelope define by local minima is zero.

Because of oscillation data, the first condition is necessary. To get the symmetric of upper and lower envelope of IMF, the second condition is required [9].

3.2 Empirical Mode Decomposition

Huang [1] invented the empirical mode decomposition method to break up original data into series of IMF, the concept is to split data into slow varying local mean piece and fast varying symmetric oscillation piece, the oscillation piece became the IMFs and the local mean the residue.

Then one can use the residue as input for further decomposition, this process repeat until no more oscillation can be separated. Because of the upper and lower envelope of IMFs is unknown initially, so on each step of decomposition one need approximate the envelope by using cubic spline function passing through the extrema of IMFs.
Then one takes the data function as initial IMF, and the refining IMFs is calculated as difference between the previous IMF and mean of the envelope, until stope condition. This process is repeat, the residue then is difference between data and improved IMF [9].

3.2.1 Summary of the EMD algorithm

Its algorithm to decomposition signal based on a sequential elimination of primary signals: the IMFs, given any signal x(t). The IMFs are organizing by a refined steps called sifting algorithm, that is summarized as follows [2]:

1. For any specific data, x(t), one can recognize all the local extrema.
2. Individually link all the minima and maxima with natural cubic spline lines to form the lower, lo(t), and upper, up(t), envelopes.
3. Hit the mean of the envelopes as me(t) = [up(t) + lo(t)] / 2.
4. Find the difference between the data and the mean as the Candidate-IMF, h(t) = x(t) - me(t).
5. Review the Candidate-IMF against the definition of IMF and the stoppage criterion to set if it is an IMF.
6. If the Candidate-IMF doesn't accept the definition, repeat step 1 to 5 on h(t) as much as needed until it accepts the definition.
7. If the Candidate-IMF does accept the definition, allocate the Candidate-IMF as an IMF component, c(t).
8. Iterate the step 1 to 7 on the residue, r(t) = x(t) - c(t), as

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the data.

9. The process ends when the residue include no further than one extremum.

3.2.2 Drawbacks of the EMD

In spite of that EMD is efficient algorithm, but it's also has some drawback, one can list as follow [10]:

- Difficulty to determine the locations of extrema.
- Because of using interpolation some unwanted overshot was introduced in extrema.
- The different stoppage condition value makes the result unpredictable.
- The criterion of the sifting stopping.
- IMF elimination.

3.2.3 An AM/FM modulation and EMD

In the shifting process, the initial step locates two sets of points that hold samples of two discrete time signals, the interpolated signals accord rating of the lower and upper envelopes, If the envelopes were symmetric one could say that x(t) is an AM signal. So, the sifting procedure is a repeated way of eliminate the dissymmetry between the lower and upper envelopes in ordered to convert the original signal into an AM signal.

So, one can infer that each IMF is an AM signal. Furthermore, as the instantaneous frequency can shift from instant to instant, one can say that each IMF is an amplitude/frequency modulated (AM/FM) signal, and the EMD is nix else than a decomposition into a group of AM/FM modulated signals.

Then, one can word that the bandwidth of the envelopes ought to be a portion of the central frequency of x(t). This means that when implementing the sifting, the low frequency components was eliminated, and leaving a high frequency signal.

This demonstrate why the IMFs appear in a high to low frequency order and why the EMD is basically a time-frequency decomposition [10].

In figure (2) below one can see how the IMF from an arbitrary signal was obtained [5].

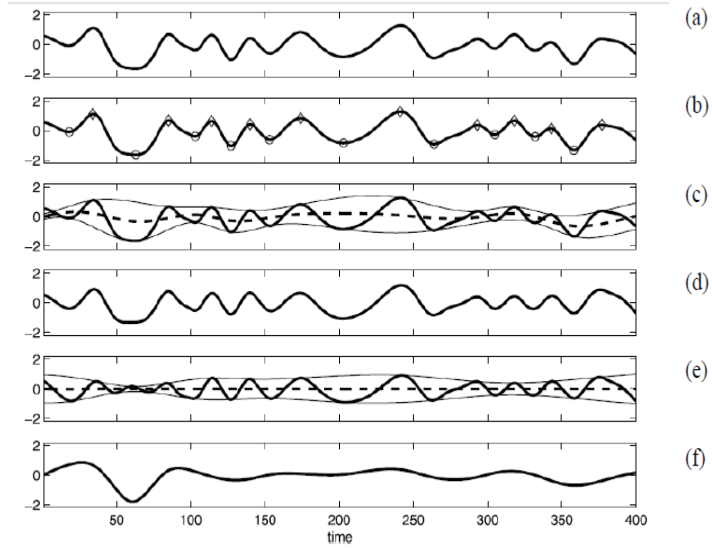

Fig. 2. The shifting process of EMD where:
a) An arbitrary input.
b) Identified maxima (diamonds) and minima (circles).
c) Upper envelope and lower envelope (solid lines) and their mean (dashed line).
d) Intrinsic mode function (IMF) (the difference between the solid line and the dashed line in (c)), that is to be re-fined.
e) Upper envelope and lower envelope and their mean of a re-fined IMF.
f) Remainder after an IMF is subtracted from the input.

As we see in figure (2-e) how we can obtain first IMF, and it's must include the finest scale or the shortest-interval oscillation in the signal, which can be extracted from the data by [5]:

$$x(t) - imf\,1 = r1 \quad . \qquad (2)$$

The residue, r1, still contains longer-interval variations, as shown in Figure (2-d). This residual is then addressed as new data and submit to the same sifting process as outlineup to gain an IMF of lower frequency, this can be frequently used to all following r, and the result is [5]:

$$r1 - imf\,2 = r2 \quad . \qquad (3)$$

$$r_{n-1} - imf_{n} = r_{n} \quad . \qquad (4)$$

The decomposition process lately halts when the residue, rn, be a function with only one extremum and no extra IMF can be taken away. So, one can have the final equation of decomposition is [5]:

$$X(t) = \sum_{j=1}^{n} imf(j) + r_n \qquad (5)$$

Thus, the original data are decomposed into n IMFs and a residue gained, rn, like an example in figure (3) [11].

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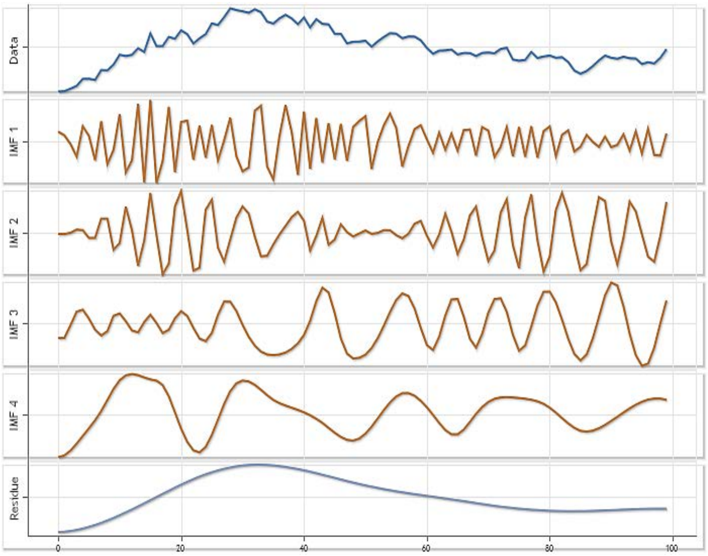

Fig. 3. Audio sample decomposed using the EMD.

3.3 Advantage of HHT

- The adaptive bases commonly satisfy all mathematical requirements for time decomposition method. Inclusive, completeness, uniqueness, orthogonality and convergence. And as we know that each IMFs specify an adaptive foundation because its gained by sifting processes [1].
- The HHT provides an alternative, and possibly more physically meaningful, representation of data [5].
- Many tests explained empirically that HHT is a sturdy tool for time-frequency analysis of nonstationary and nonlinear data. It is instituted on an adaptive basis. So, HHT is better than Fourier because Fourier transform is limited only to linear system and stationary data series, and it's better than Wavelet because wavelet transform function it's also limited to linear system and fixed length [11].

One can see in the table (1) the compression between Fourier, Wavelet, and HHT [5].

TABLE 1
COMPRESSION BETWEEN FOURIER, WAVELET AND HHT.

FEATURE	FOURIER	WAVELET	HHT
BASIS	a priori	a priori	adaptive
FREQUENCY	convolution over global domain, uncertainty	convolution over global domain, uncertainty	differentiation over local domain, certainty
PRESENTATION	energy in frequency space	energy in time-frequency space	energy in time-frequency space
NONLINEARITY	No	Yes	Yes
NONSTATIONARY	No	No	Yes
FEATURE EXTRACTION	No	discrete, no; continuous, yes	Yes
THEORETICAL BASE	complete mathematical theory	complete mathematical theory	empirical

4. APPLICATION OF HHT

One can describe here some of HHT application in different domain:

A Digital Watermarking

In [11] a novel audio watermarking algorithm depended on EMD is proposed. The samples of audio first split into number of frames and then each frame by using EMD decomposed into IMFs intrinsic components. Then the data (watermark) insert into the last IMF, exactly in extrema point. Before one can insert data, its merge with the synchronization codes. This method shows the robustness against additive noise, re-quantization, MP3 compression, filtering, resampling and cropping.

B Biomedical Applications

In [12] a novel technique for electroencephalogram (EEG) signals analysis, which using EMD and Fourier-Bessel (FB). First the EEG signal decomposed into a fixed group of intrinsic signals named IMFs by using EMD. Then for each IMF, one can calculate MF (mean frequency), by using FB. So one can find the difference between seizure-free and ictal EEG signals, depended on MF measure.

C Image Processing

In [14] a method for image fusion and enhancement, using EMD. In this method images are decomposed, instead of signals into their intrinsic IMFs. At the decomposition level, fusion is applied. And one can rebuilding the fused IMFs to see the fused image. Between IMFs, there are a weighting plans that lessening the mutual information. that way raising the visual content and information of fused image.

D Health Monitoring

In [15] a technique based on EMD have been applied for gear fault diagnosis in mechanical systems. The EMD decomposes an original signal into various frequency-bands in time domain, famous as IMFs. Also a cosine window-based method has been applied, when the midst component of it can be decomposed into IMFs. The cosine method does only for a special IMF relying on the size of window. The experimental results and simulation for this technique show that is a sturdy and reliable technique for fault diagnosis.

E Speech Recognition

In [16] a novel pitch determination technique based on EMD, when the speech signal is decomposing into a group of IMFs by using EMD. The instantaneous frequency and amplitude of all IMFs are obtained using HT. After this post processing is used to define the essential frequency or pitch period of speech data.

F Iris Recognition

In [13] a novel analysis for iris recognition technique based on HHT, when this proposed method first divides a normalize image (iris), into sub-regions. Then for every sub-region utilize the center mean frequency information to the vector of feature. This suggest technique for iris recognition has some good properties, such as rotation invariance, translation invariance, scale invariance, and also show robustness to noise that has high frequency. Furthermore, the experimental result display that the showing of this suggest method is similar to the top algorithm of iris recognition found in recent.

5 Conclusions

HHT show a technique that can deal with nonstationary and nonlinear data analysis, mostly for representations of the time frequency energy. It has vastly try out in different applications. Also HHT accord results preferable than of the classical techniques in different case. and in most statues, it makes the technique more robust by showing true physical meanings of it. We also review the use of the HT for spectral estimation, and how it's important in HHT process. The optimization to the problems that related with HHT, such as the election of spline, the position of extrema and end effect, etc. It can be nice discussed area in a future research.

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